Indian Statistical Institute
Semestral Examination
Topology IV - MMath II

Max Marks: 60 Time: 180 minutes.

Throughout, U, $V \ldots$ will denote open subsets in some Euclidean space.

(1) (a) Show that
$$\wedge : \mathrm{Alt}^1(\mathbb{R}^3) \times \mathrm{Alt}^1(\mathbb{R}^3) \longrightarrow \mathrm{Alt}^2(\mathbb{R}^3)$$
is surjective. Is
$$\wedge : \mathrm{Alt}^1(\mathbb{R}^4) \times \mathrm{Alt}^1(\mathbb{R}^4) \longrightarrow \mathrm{Alt}^2(\mathbb{R}^4)$$
surjective? Justify. [10]

(b) Compute $H^0(U)$. [10]

(2) (a) Let $U = \mathbb{R}^2 - \{(x,0) : x \leq 0\}$ and $V = \mathbb{R}^2 - \{(x,0) : x \geq 0\}$. Find a basis of $H^0(U \cap V)$ and $H^1(U \cup V)$. Find the matrix of the linear map
$$\partial : H^0(U_1 \cap U_2) \longrightarrow H^1(U_1 \cup U_2)$$
with respect to these basis. [10]

(b) Let $\Sigma \subseteq \mathbb{R}^3$ be a knot. Compute $H^p(\mathbb{R}^3 - \Sigma)$, $p \geq 0$. [10]

(3) (a) Show that $\mathbb{R}P^{n-1}$ is orientable if and only if n is even. [10]

(b) Let F be a vector field defined on V and let $x \in V$ be an isolated singularity of F. Define the index of F at x. Let F be the vector field $F(x,y) = (y,x)$ on $\mathbb{R}^2$. Compute the index of F at $(0,0)$. [10]